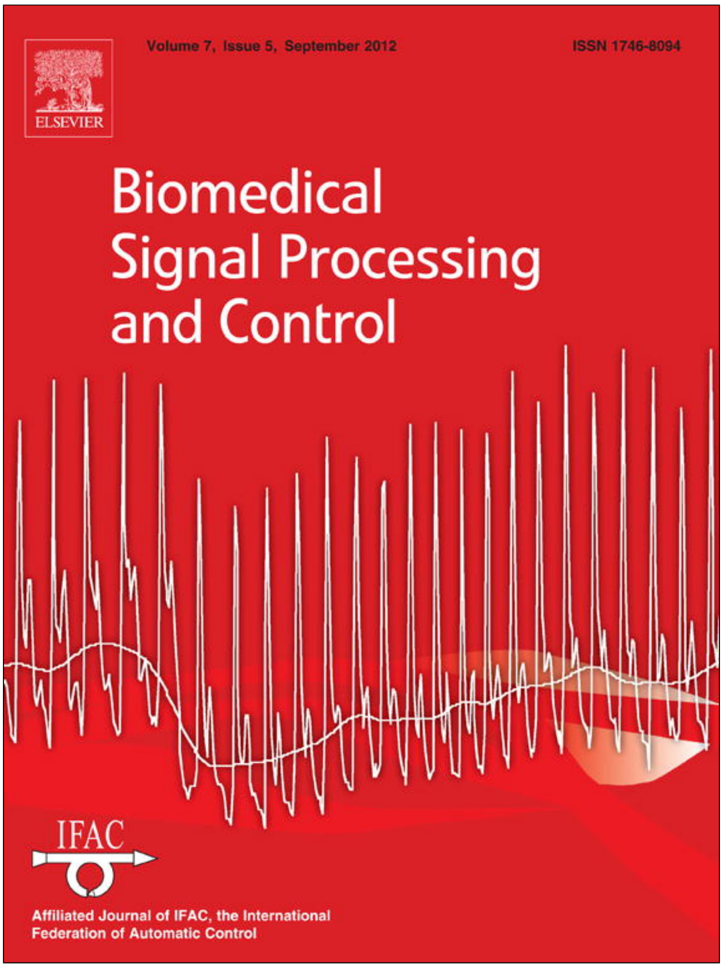
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Technical note

Estimation of the noise autocorrelation function in auditory evoked potential applications

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ABSTRACT

This work presents a simple and accurate method to estimate the noise autocorrelation function in auditory evoked potential applications. It basically consists in applying a conventional correlation function estimator over the contaminated evoked potential signal processed by a comb filter. The main feature of the proposed technique is the possibility of obtaining information on large correlation lags without the need of extra time intervals, minimizing the estimation time. A theoretical analysis is provided showing that, under certain but achievable conditions, the correlation function of the processed signal approximates the real noise correlation function. Simulation results and an example with a single-trial evoked potential estimation technique illustrate the expected performance. The proposed method is of special interest to either single or small number of trials evoked potential estimation techniques in anaesthesia monitoring applications.

1. Introduction

Evoked potentials (EPs) are electrophysiological responses of the nervous system to sensory stimuli. The interest in these biopotentials resides in their importance to the clinical diagnosis of several pathologies and in the study of brain functions. EPs are usually measured through surface electrodes positioned at particular locations in the scalp. This measurement setup leads to contamination of the signal of interest by background spontaneous electroencephalographic activity, myoelectric signals and external interferences. As a consequence, the raw acquired EP usually presents a very low signal to noise ratio (SNR) typically ranging from −10 dB down to −30 dB [1]. Thus, it is important to improve the EP quality before clinical interpretation. Unfortunately, classical filtering techniques do not apply – since the EP and noise share the same spectral bandwidth – while weighted ensemble averaging techniques can incur in biasing [2]. Hence, the conventional ensemble averaging (EA) is still the most used technique to estimate EPs.

Conventional EA consists of a synchronous averaging of single sweeps time-locked to external stimuli. This technique is based on the assumptions that EP is a periodic deterministic signal[1] and that contamination noise is a zero mean uncorrelated signal.

Important practical applications are characterized by progressive and significant variations of the EP waveform along the time. This is the case of anaesthetic monitoring, where amplitude and latency changes of the midlatency auditory evoked potentials (MLAEP) present a direct relation with anaesthetic depth [3–5]. Therefore, important information may be lost when large amounts of data are averaged. Thus, it is of interest to obtain EP estimates with the smallest number of sweeps.

In general, MLAEP extraction requires averaging of hundreds of sweeps to obtain a suitable estimate, due to the MLAEP very small SNR nature. This approach results in long estimation times, which is especially troublesome in real-time monitoring applications. However, during anesthetized states MLAEP SNR increases significantly [4] and small or even single-trial estimation techniques could be used during unconscious states to provide faster and more accurate predictions.

Many techniques have been proposed to enhance the quality of EP estimations obtained by a small or a single number of sweeps. Examples are the Wiener filtering of small-averaged EPs [6],

[1] Recent studies have supported the stochastic nature of EPs, even in normal individuals under rest (due to fatigue, habituation or level of attention), that are associated with different states of brain function [11]. In this work, these EP variations have been neglected since it is assumed that they have no clinical interest in the application at issue and the magnitude and latency variations are minimal when compared to those of interest.

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subspace techniques [7] and particle filtering [8]. The main drawback of such techniques is the required knowledge of the signal and noise statistics.

In practice, EP statistical information is obtained from small-averaged signals, while noise information is obtained from measurements before stimuli [8–11]. This noise estimation method, however, requires a large time between stimuli. There are two reasons for that: a needed period for EP late potential vanishing and (most important) the availability of a sufficient number of EP-free samples for accurate estimation. This is especially troublesome when large correlation lags must be estimated. A simple and accurate noise statistics estimator was proposed in [12]. Unfortunately, it demands long intervals between stimuli for proper estimation of large correlation lags (in order to minimize biasing due to noise autocorrelation). In addition, this estimator has been proved unbiased only in the unreal condition of uncorrelated noise. As a result, it provides good results only for small correlation lags. Nevertheless, many EP estimation techniques that require knowledge of the noise autocorrelation function have been addressed by scientific literature in recent years [7–14]. An extensive search in the main scientific sources shows that no other methods for estimating the noise autocorrelation function in EP applications have been published since [12], resulting in the need of large inter-stimuli periods or restricting the application to odd-ball tests,[2] such as in P100 and P300 estimation, where long EP-free periods are available. Given the difficulties in estimating the noise correlation, even some recent works have arbitrarily assumed, in performance tests, the unreal condition of uncorrelated background noise [15], losing the opportunity of taking advantage of signal information. As a result, this subject is an open state-of-the-art issue in present-day literature in the area.

This work presents a simple and reliable method for estimating the autocorrelation function of the background noise in auditory evoked potential applications. The proposed method does not require extra intervals between stimuli, resulting in accurate and fast estimates even for large correlation lags. This work is particularly relevant in situations of medium to high SNR as in unconscious states during anaesthetic monitoring.

2. Comb filtering

Assuming the scalp measurement in a given period of time can be described as $x(n) = s(n) + \upsilon(n)$, where $x(n)$ is the sampled measurement at time n, $s(n)$ is the EP, and $\upsilon(n)$ is the background noise (mainly characterized by EEG), we define the filtered measurement $y(n)$ as:

$$y(n) = \frac{1}{\sqrt{2}}[x(n) - x(n-\Delta)] \tag{1}$$

Eq. (1) implements a comb filter with notches at frequencies F_{samp}/Δ; where F_{samp} is the sampling frequency, $\Delta = \kappa N_E$, N_E is the total number of samples of each epoch, and κ is a positive integer. We can interpret it as the synchronous subtraction of two κ-separated sweeps.

Assuming EP variations can be disregarded in a time-window of κ consecutive sweeps, the filtered measurement signal is an EP-free signal whose properties are inherited from the background EEG and by the characteristics of the comb filter. The next sections will show that, under certain conditions, an estimate of the EEG background autocorrelation function can be obtained directly from an estimate of the filtered measurement autocorrelation function, independently of the characteristics of the comb filter.

[2] The odd-ball test consists in obtaining EPs while an individual is asked to perform a specific task when an infrequent stimulus (target) is applied between non-relevant stimuli (standard).

3. Estimation of the noise correlation function

Two different approaches can be used for obtaining estimates of the autocorrelation function of a stochastic signal [16]: (a) parametric solutions based on time series models and (b) the average of lagged-products of the observations. Besides their great accuracy, methods based on the first approach can require large computational resources, being time-consuming. The most known member of the second family is the sample autocorrelation function and its variations. Its main appeal is its computational complexity that permits real-time estimations. Its main drawback is that it is a non-stationary stochastic process with mean and variance that are functions of the lag distance (a biased solution). However, useful estimates can be obtained when the number of samples is large (greater than 50) and the maximum lag is less than one-quarter of the available data points [17]. The sample autocorrelation function of the measurement signal is defined as [18]:

$$\hat{F}_y(k) = \frac{1}{N}\sum_{i=1}^{N-k} y(i)y(i+k) \tag{2}$$

where $y(n)$ is defined in (1) and assumed ergodic; $0 \le k < P$; P is the maximum lag of interest, N is the number of available samples and $P < N/4$. Eq. (2) is evaluated at each sweep using N past samples of the filtered measurement A strictly unbiased solution can be obtained if the scale factor in the denominator of Eq. (2) is changed to $(N-k)$. However, the estimated function for all lags k is not assuredly positive-definite.

4. Theoretical analysis

The proposed noise autocorrelation function estimator is obtained by filtering the EP signal by Eq. (1) and by evaluating Eq. (2). In this section, it is showed that such procedure results in an unbiased estimation of the true noise autocorrelation function, whose variance decreases to zero as the number of available samples (N) tends to infinity, *i.e.* the larger the number of available samples, the better the estimate.

Assuming a time interval $\Delta \cdot T_{samp}$ (where $T_{samp} = 1/F_{samp}$ is the sampling period) in which the EP does not present significant variations ($s(n) \cong s(n-\Delta)$), the filtered measurement $y(n)$ is free from EP influence and Eq. (1) turns to:

$$y(n) = \frac{1}{\sqrt{2}}[\upsilon(n) - \upsilon(n-\Delta)] \tag{3}$$

Using (3) in (2) and taking the expected value (defined as $E\{\cdot\}$) we obtain:

$$E\{\hat{F}_y(k)\} = \frac{1}{2N}\sum_{i=1}^{N-k} E\{\upsilon(i)\upsilon(i+k)\} - \frac{1}{2N}\sum_{i=1}^{N-k} E\{\upsilon(i)\upsilon(i+k-\Delta)\}$$
$$- \frac{1}{2N}\sum_{i=1}^{N-k} E\{\upsilon(i-\Delta)\upsilon(i+k)\} + \frac{1}{2N}\sum_{i=1}^{N-k} E\{\upsilon(i-\Delta)\upsilon(i+k-\Delta)\} \tag{4}$$

Defining the noise autocorrelation function as:

$$F_\upsilon(b-a) \equiv E\{\upsilon(i-a)\upsilon(i-b)\} \tag{5}$$

and using (5) in (4) it results in:

$$E\{\hat{F}_y(k)\} = \frac{N-k}{N}F_\upsilon(k) - \frac{N-k}{2N}F_\upsilon(k-\Delta) - \frac{N-k}{2N}F_\upsilon(k+\Delta) \tag{6}$$

Considering $\Delta > 2k_{\max}$, where $k_{\max}$ is the noise correlation length (a number that results in $F_v(k_{\max}+1) < \varepsilon$ where ε is very small, so as $F_v(k_{\max}+1)$ can be assumed zero), then

$$E\{\hat{F}_y(k)\} \cong \frac{N-k}{N} F_v(k) \tag{7}$$

Assuming large N and $P < N/4$, where $0 \le k < P$, then

$$E\{\hat{F}_y(k)\} \cong F_v(k) \tag{8}$$

Eq. (8) shows that Eqs. (1) and (2), under some specific but achievable practical conditions, provide an unbiased estimator of the noise autocorrelation function. Better accuracy can be obtained from the strength of the used assumptions.

4.1. Analysis of the estimator's consistency

A first order statistical analysis of an estimator cannot guarantee consistency[3] and a second order analysis is needed to demonstrate the usefulness of the estimator in practical applications [12]. The variance (defined as var{·}) of the estimator is given by [18]:

$$\begin{aligned} \mathrm{var}\{\hat{F}_y(k)\} &= E\{[\hat{F}_y(k) - E\{\hat{F}_y(k)\}]^2\} \\ &= E\{\hat{F}_y^2(k)\} - E\{\hat{F}_y(k)\}^2 \end{aligned} \tag{9}$$

Squaring (2), taking its expected value, assuming Gaussian noise and using (7) in (9), after some arithmetic, it comes to

$$\begin{aligned} \mathrm{var}\{\hat{F}_y(k)\} = & \frac{(N-k)^2}{4N^2}[F_v^2(k+\Delta) + F_v^2(k-\Delta)] \\ & + \frac{(N-k)^2}{2N^2} F_v(k-\Delta)F_v(k+\Delta) \\ & - \frac{(N-k)^2}{N^2} F_v(k)[F_v(k+\Delta) + F_v(k-\Delta)] \\ & + \frac{1}{N^2}\sum_{i=1}^{N-k}\sum_{j=1}^{N-k} F_v^2(j-i) \\ & + \frac{1}{N^2}\sum_{i=1}^{N-k}\sum_{j=1}^{N-k} F_v(j-i+k)F_v(j-i-k) \\ & + \frac{1}{2N^2}\sum_{i=1}^{N-k}\sum_{j=1}^{N-k} F_v(j-i-\Delta)F_v(j-i+\Delta) \\ & + \frac{1}{4N^2}\sum_{i=1}^{N-k}\sum_{j=1}^{N-k} [F_v^2(j-i+\Delta) + F_v^2(j-i-\Delta)] \\ & - \frac{1}{N^2}\sum_{i=1}^{N-k}\sum_{j=1}^{N-k} F_v(j-i)[F_v(j-i+\Delta) + F_v(j-i-\Delta)] \\ & + \frac{1}{4N^2}\sum_{i=1}^{N-k}\sum_{j=1}^{N-k} F_v(j-i+k-\Delta)[F_v(j-i-k-\Delta) \\ & + F_v(j-i-k+\Delta)] \\ & + \frac{1}{4N^2}\sum_{i=1}^{N-k}\sum_{j=1}^{N-k} F_v(j-i+k+\Delta)[F_v(j-i-k-\Delta) \\ & + F_v(j-i-k+\Delta)] \\ & - \frac{1}{2N^2}\sum_{i=1}^{N-k}\sum_{j=1}^{N-k} F_v(j-i+k)\,[F_v(j-i-k+\Delta) + F_v(j-i-k-\Delta)] \\ & - \frac{1}{2N^2}\sum_{i=1}^{N-k}\sum_{j=1}^{N-k} F_v(j-i-k)[F_v(j-i-k+\Delta) + F_v(j-i-k-\Delta)] \end{aligned} \tag{10}$$

Assuming the noise autocorrelation function is finite ($F_v(k) < \varepsilon$ for $-k_{\max} > k > k_{\max}$, $\varepsilon \to 0$) and $\Delta > 2k_{\max}$, the three first terms in (10) are zero. The remaining terms consist of a sum of a maximum of $N{\cdot}k_{\max}$ values divided by N^2. This way, variance tends to zero as the number of available samples tends to infinity:

$$\lim_{N\to\infty} \mathrm{var}\{\hat{F}_y(k)\} \to 0 \tag{11}$$

The statistical assumptions necessary to demonstrate the validity of the proposed estimator are no more restrictive than those assumed for the EA technique (EP deterministic and non-correlated EEG).

5. Real-time implementation

Based on the theoretical results presented in previous sections, it can be inferred that, assured the needed assumptions, different approaches of time-average lagged-products of the observations can be used to estimate the noise autocorrelation function from the filtered measurement signal. A low-cost real-time estimation method can be implemented by a vector accumulator and a delay-line ($\mathbf{y}(n) = [\, y(n) \quad y(n-1) \quad \ldots \quad y(n-M+1)\,]^T$) as:

$$\hat{\mathbf{f}}_y = \frac{1}{N-M}\sum_{n=M}^{N-1} y(n)\mathbf{y}(n) \tag{12}$$

where $\hat{\mathbf{f}}_y = \left[\hat{F}_y(0) \quad \hat{F}_y(1) \quad \ldots \quad \hat{F}_y(M-1)\right]^T$ is a vector containing the lagged products of the estimated autocorrelation function,[4] N is the number of available samples in the time interval where the autocorrelation sample is assumed to be evaluated, and $\mathbf{y}(n)$ is updated for $n = 0$ to N. The real-time algorithm, given by Eqs. (1) and (12), presents a computational complexity of $(M+1)$ multiply-accumulate operations per sample, requiring $(\Delta + 2M)$ memory positions. Eq. (12) can also be recursively implemented. In case of a continuous estimation of the autocorrelation function, a computationally efficient alternative is

$$\hat{\mathbf{f}}_y(n) = \alpha\hat{\mathbf{f}}_y(n-1) + (1-\alpha)y(n)\mathbf{y}(n) \tag{13}$$

where $(1-\alpha)$ is a small constant.[5] Eq. (13) presents a computational complexity of $(2M+1)$ multiply-accumulate operations per sample.

6. Simulation results

In order to verify the performance of the proposed method, it was initially compared with the method presented in [12] in two controlled environments, consisting of: (1) stationary noise added to a known replicated real EP and (2) nonstationary noise added to a simulated variable EP (simulating EEG and EP variations due to an anaesthetic procedure). The results obtained by Eqs. (2) and (12) and Eq. (21) from [12] for different numbers of epochs (an epoch contains T samples) were compared with the true noise autocorrelation function (derived from the AR model) following three figures of merit to be later defined. Finally, the proposed method was applied in conjunction with a single-trial subspace technique in order to demonstrate its usefulness in practical EP estimation applications without intervals between stimuli.

In all simulations the estimated correlation function and the EP epoch had the same length ($M = N_E$) of 601 samples (0,12s). This length permits detection of brainstem auditory evoked potentials and perfect analysis of midlatency waves. The delay parameter κ was set to 2. Monte Carlo experiments indicate that values of κ up

[3] A statistical estimator is defined as being consistent when its variance tends to zero as the number of samples tends to infinity. This characteristic guarantees that a lower variance of the estimate will be obtained with the increase of the number of available samples.

[4] For a detailed analysis and properties of (12) refer to [19] (Chapter 5.2).

[5] Eq. (13) is a first order discrete low-pass filter with a time constant given by $\tau = -T_{amos}/\ln(\alpha)$ seconds [23]. The time constant defines the time required for the influence of the input to vanish to 37% of its initial value.

to 10 can be used without noticeable loss in accuracy. All real data (evoked potential and electroencephalographic signals) presented in this work were obtained from a previously recorded database and here were off-line processed in Matlab.

Example 1. *Estimation of the noise autocorrelation function assuming deterministic EP and stationary noise*: the known evoked potential (Fig. 1(i)) was obtained by averaging 1000 sweeps of a real auditory evoked potential signal acquired from a normal subject under rest [20]. The simulated noise was computer-generated from a 16th order autoregressive (AR) model whose coefficients were estimated via Burg's method from real EEG sampled at 5 kHz (specialized literature indicates that an order of 5 is sufficient for a 5% accuracy and an order of 10 or higher can lead to optimal accuracy [21]). The resulting AR model impulse response magnitude is reduced to less than 1% of the first sample magnitude after 88 ms (440 samples). The EP signal and simulated noise were added and the SNR was determined at −23 dB. Fifty realisations were performed. Results are presented in Fig. 1.

Example 2. *Estimation of the noise autocorrelation function assuming nonstationary EP and nonstationary noise*: the nonstationary EPs were mathematically designed according to the procedure described in [4]. It resulted in a sequence of artificial midlatency potentials with a rate of change compatible with pharmacokinetic time constants of some usual anaesthetic agents. The synthetic signal is described by a damped sinusoid ($s(n) = 100d^n \sin(2\pi fn + \phi)$) with parameters adapted from [4] to the sampling frequency of 5 kHz. These parameters were linearly changed over time to simulate the pre ($d = 0.998$; $f = 0.0046$; $\phi = -0.001$) and full ($d = 0.9926$; $f = 0.0034$; $\phi = -2.02$) anaesthesia stages. The SNR varied from −20 dB (conscious state) to 10 dB (anaesthetized state) using the same AR model described in Example 1, but with a nonstationary power (linear variation). Such SNR variation is characteristic of the induction process during anaesthesia. The resulting signal mimics a time between the conscious state to the full anaesthetized one of about 4 min (Fig. 2(i)). Fifty realisations were performed. Results are presented in Fig. 2.

Example 3. *Application with a single-trial EP estimation technique*: in order to demonstrate the usefulness of the proposed method in a practical application, it was used in conjunction with the subspace method for single-trial estimation of evoked potentials presented in [7]. This method uses second-order statistics to form a prior information model for the evoked potentials. It was originally applied to the P300 test, which supplies long periods without evoked potentials and, consequently, allows the estimation of noise second-order statistics directly from the unprocessed signal. The authors of [7] suggested the use of parametric methods (that need large computational capacity) or the prior assumption of a white noise statistics (identity correlation matrix) in case large variations in the statistics of the background noise are obtained (which occurs when the inter-stimulus period is not large enough). However, such assumption is far-fetched and its use may significantly impair the estimation technique performance. As a result, the method presented in [7] has a limited application, once even other kinds of large amplitude evoked potentials, such as steady-state evoked potentials, will require large inter-stimuli periods in order to estimate the needed statistics accurately. In this example, with the help of the proposed technique, the applicability of [7] was extended to the case of auditory evoked potentials without inter-stimuli periods. Here, unlike Examples 1 and 2, we evaluated the difference between the clean EP (the real deterministic evoked potential presented in Fig. 1(i)) and its estimation from the noisy EP (the clean EP contaminated by the simulated noise described in Example 1) by using: (a) the unprocessed noisy EP; (b) the noisy EP processed by the technique presented in [7] assuming the noise is white; and (c) the noisy EP processed by the technique presented in [7] combined with the proposed method in which the noise second order statistics were continuously estimated through Eq. (12) using only 2 epochs (present and past). Simulation was performed with 1000 epochs and SNR ranged from −25 dB to 5 dB with steps of 2 dB.

Three figures of merit were used to evaluate the performance of the methods under comparison: (a) estimation error (*ER*); (b) relative error (*RE*); and (c) spectral distortion (*SD*).

The *ER* is a standard quality measure defined by the following equation:

$$ER = \frac{1}{R}\sum_{r=1}^{R} \frac{\sum_{k=0}^{M-1}[\bar{F}_v(k) - (\hat{F}_{yr}(k))/(\hat{F}_{yr}(0))]^2}{\sum_{k=0}^{M-1}\bar{F}_v^2(k)} \tag{14}$$

where R is the number of the available stochastic realisations of the contamination noise, M is the length of the autocorrelation function, $\bar{F}_v(k) = Fv(k)/Fv(0)$ is the kth lag of the normalized theoretical noise autocorrelation function and $\hat{F}_{yr}(k)$ is the kth lag of the estimated autocorrelation function obtained in the rth realisation. It measures the mean square difference between the theoretical autocorrelation function and the estimated one for a given number of sweeps. In this example, noise power estimation is not under consideration since there is a variety of simple estimators that can result in accurate estimations.

The relative error is defined as:

$$RE_l(k) = \left| \frac{\bar{F}_v(k) - \frac{1}{R}\sum_{r=1}^{R}(\hat{F}_{y_{l,r}}(k))/(\hat{F}_{y_{l,r}}(0))}{\bar{F}_v(k)} \right| \tag{15}$$

where R is the number of the available stochastic realisations of the contamination noise, $\hat{F}_{y_{l,r}}(k)$ is the kth lag of the estimated autocorrelation function obtained with the first l sweeps in the rth realisation and $|\cdot|$ is the absolute value. It permits to verify the individual relative errors (of each autocorrelation lag) for a given number of sweeps.

Finally, spectral distortion is defined as the cepstral mean square error between the estimated and theoretical autocorrelation functions. It is used to evaluate the coherent structure of the autocorrelation function instead of its individual points [22]. The spectral distortion is defined as [16]:

$$SD = \frac{1}{R}\sum_{r=1}^{R}\frac{1}{4\pi}\sum_{\omega=0}^{M-1}\{\ln[h(\omega)] - \ln[\hat{h}_r(\omega)]\}^2 \tag{16}$$

where R is the number of the available stochastic realisations of the contamination noise, $h(\omega)$ is the ωth element of vector $\mathbf{h} = |F\{\mathbf{f}_v/F_v(0)\}|$; $\mathcal{F}\{\cdot\}$ is the fast Fourier transform (FFT) of its argument; $F_v(0)$ is the first element of vector $\mathbf{f}_v = \begin{bmatrix} F_v(0) & F_v(1) & \ldots & F_v(M-1) \end{bmatrix}^T$ that contains the true elements of the noise autocorrelation function; $\hat{h}_r(\omega)$ is the ωth element of vector $\hat{\mathbf{h}}_r = |F\{\hat{\mathbf{f}}_{yr}/\hat{F}_{yr}(0)\}|$; $\hat{F}_{yr}(0)$ is the first element of vector $\hat{\mathbf{f}}_{yr} = [\,\hat{F}_{yr}(0) \quad \hat{F}_{yr}(1) \quad \ldots \quad \hat{F}_{yr}(M-1)\,]^T$ that contains the lags of the estimated autocorrelation function obtained in the rth realisation.

For Example 3, the autocorrelation function and its estimate described in the definition of (14–16) were substituted by the clean EP and its estimate, respectively. The results are presented in Figs. 1–3.

7. Discussion

Examples 1 and 2 provided means of demonstrating the validity and performance of the proposed method related to [12].

Fig. 1(ii) shows the *ER* index as a function of the number of sweeps for the estimator proposed in [12] and estimators described

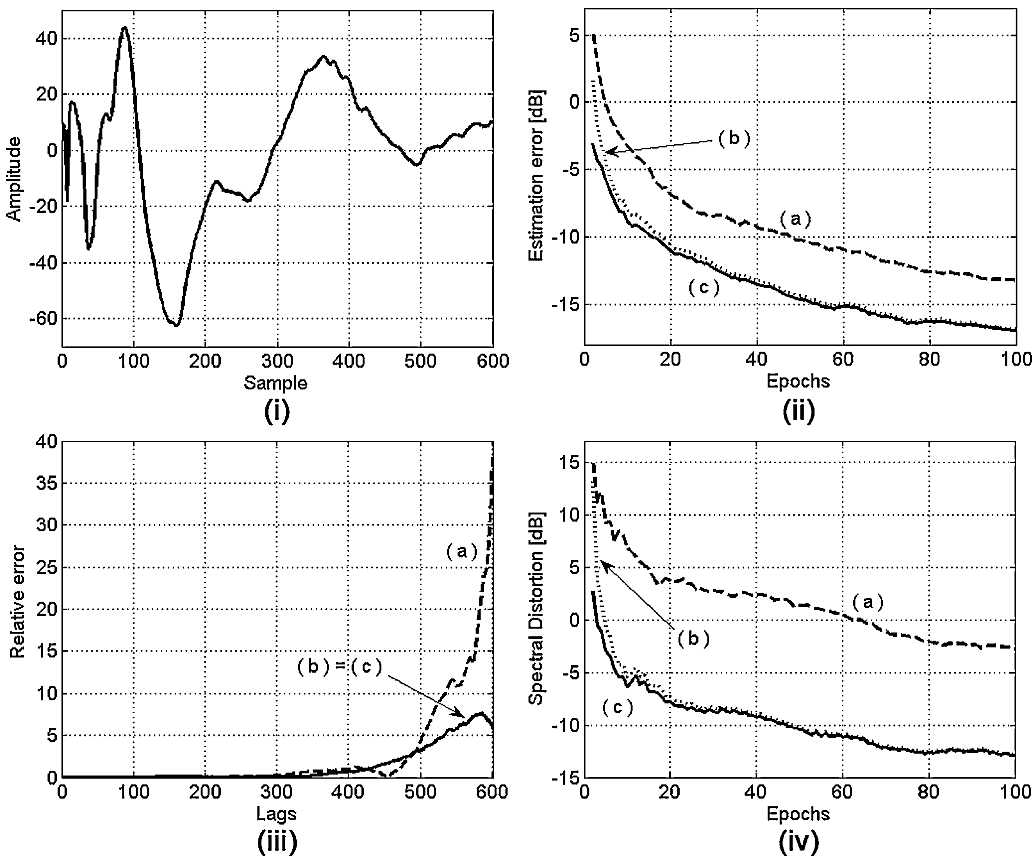

Fig. 1. Example 1: SNR = −23 dB. (i) Real evoked potential; (ii) estimation errors; (iii) relative errors; (iv) cepstral estimation errors. Estimator presented in: (a) Ref. [12] (dashed line); (b) Eq. (12) (dotted line); and (c) Eq. (2) (solid line).

by Eqs. (2) and (12) in the stationary environment described in Example 1. It is possible to verify that Eqs. (2) and (12) lead to very similar results for a number of epochs larger than 5 and that both produce better estimates than the method presented in [12]. The *ER* performance gain of the proposed technique over that presented in [12] slowly reduces with the number of available trials. Calculations of the *RE* index for Example 1 are shown in Fig. 1(iii). This figure indicates that the large lags of the autocorrelation function are the main source of error for both [12] and the proposed technique. However, it is clearly seen that the proposed technique improves the estimate of large autocorrelation lags, resulting in smaller *RE* indexes.

Fig. 1(iv) presents the most important result for Example 1. It shows that the proposed technique presents a very high *SD* gain performance, when compared to [12], and such gain does not vanish with the increase of the number of available sweeps. The *SD* takes into account the relationship amongst autocorrelation lags (by using the ratio of spectral components), an important characteristic needed to preserve statistical information and spectral content of the signal under analysis [16]. Albeit both methods result in accurate estimations of the first correlation lags, the small *SD* values indicate that the proposed method produces better estimates of the whole structure of the correlation function [16]. This is especially important when using noise statistics for designing filtering or extraction procedures.

Results for the nonstationary environment described in Example 2 are presented in Fig. 2. They corroborate and expand the conclusions obtained by the stationary case in Example 1. The proposed method results in better estimates of the individual lags of the autocorrelation function. These estimates are especially improved in case of larger lags. The spectral distortion index shows the proposed method is able to produce better estimates of the structure of the autocorrelation function even in the event of non-stationary signals. This gain in *SD* performance does not vanish with the increase of sweeps. From these results, it is evidenced that fast variations on the EP and EEG, such as those related to a real anaesthetic procedure, do not significantly impact on the performance of the proposed method once only small periods of time are needed to obtain the estimates.

In Example 3, a single-trial estimation technique was applied to auditory evoked potentials in conjunction with the proposed method, assuming no availability of inter-stimuli periods. In such case, the authors in [7] suggest using an identity covariance matrix. However, it is well known that background noise in EP applications is far from being uncorrelated. As a result, the performance of the technique presented in [7] will not be optimal. In order to verify the technical contribution of our proposal, the performance of [7] was tested in two different situations: (a) the arbitrary (and erroneous) assumption of uncorrelated noise (as suggested by [7]) and (b) continuous estimation of the noise second order statistics by the proposed method. Fig. 3 presents the *SD* index for unprocessed signals, signals processed by [7] with the use of an identity covariance matrix, and processed by [7] with a covariance matrix obtained through Eq. (12). In this Example, the *SD* index is evaluated using the noise-free evoked potential as $\mathbf{h}$ and the estimated/noisy EP at epoch r as $\hat{\mathbf{h}}_r$. Clearly, the proposed technique provides improved results. For SNRs higher than 0 dB, the subspace method degrades the quality of EP once any kind of processing will distort important EP frequency components.

In recent years, many different kinds of estimation techniques to improve the quality of evoked potential signals have been developed, such as subspace methods, particle filtering and independent component analysis [7–14]. Although second-order statistics knowledge is required by most of these techniques, the

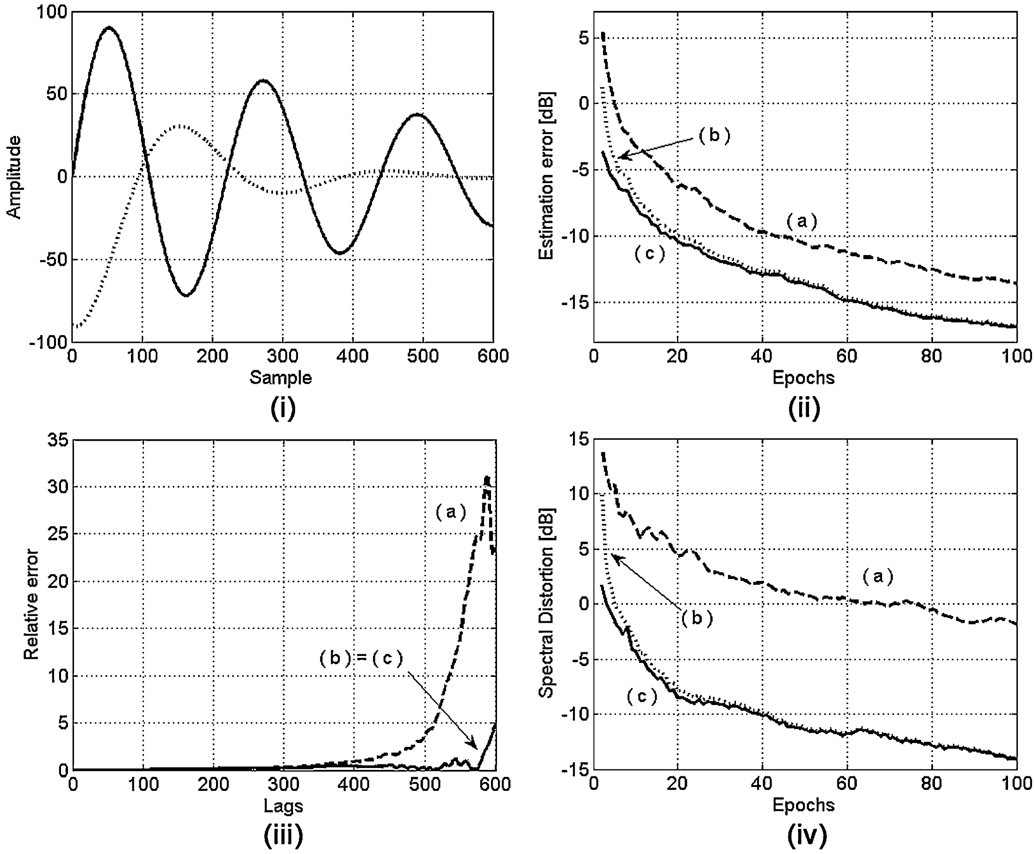

Fig. 2. Example 2: −20 dB ≤ SNR ≤ 10 dB. (i) Simulated evoked potential for conscious state (solid line) and full anaesthetized state (dotted line); (ii) estimation errors; (iii) relative errors; (iv) cepstral estimation errors. Estimator presented in: (a) Ref. [12] (dashed line); (b) Eq. (12) (dotted line); and (c) Eq. (2) (solid line). The evoked potential was continuously varied from the conscious to the full anesthetized shape (see [4]) along the 100 epochs.

method through which these statistics can be obtained is not covered in the original articles, resulting in a very restricted set of applications characterized by large inter-stimuli periods (essentially odd-ball tests). The proposed technique allows obtaining very precise estimations of the noise autocorrelation function without the need of inter-stimuli periods and with a low computational cost. This characteristic is very useful and can extend the list of applications of new estimation techniques, such as in [11] and [13] to steady-state EPs and auditory evoked potentials during anaesthetic states.

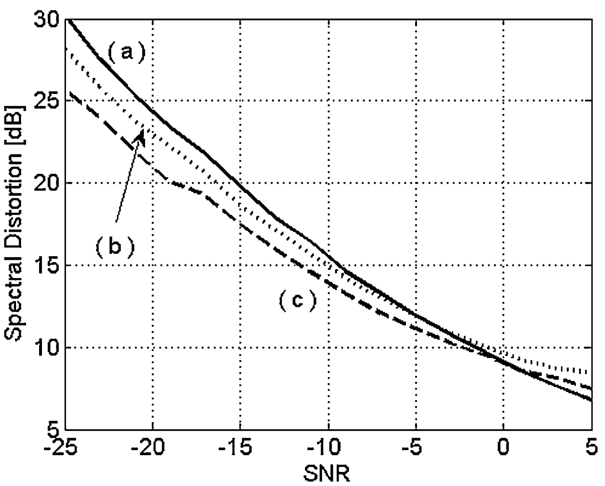

Fig. 3. Example 3: Spectral distortion of real single-trial auditory evoked potentials [20]. Simulation of 1000 epochs. (a) Raw EP (solid line); (b) EP processed by [7] assuming white signals (dotted line); and (c) EP processed by [7] using the autocorrelation function estimated by (12) (dashed line).

8. Conclusions

This work presented a new method for estimating the noise autocorrelation function in auditory evoked potential applications. Theoretical analysis demonstrated the needed conditions for obtaining unbiased and consistent estimates using time-averages of lagged-products of the observations. Simulation results have shown a performance gain over a technique previously presented in literature. The joint application of the proposed technique with a subspace single-trial EP estimation technique demonstrated the possibility of lowering spectral distortion of the estimates without the need of any inter-stimuli periods. This technique is specially recommended for joint use with evoked potential estimation techniques for a single or a small number of trials which require long correlation lags and small estimation times.

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